2021

MATHEMATICS — HONOURS

First Paper

(Module – II)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations have their usual meanings

Group - A

(Marks : 20)

Answer ***question no.*** **1** and ***any two*** questions from the rest.

1. Answer ***any one*** question :

(a) (i) For what value of λ does $\lambda xy - 8x + 9y - 12 = 0$ represent a pair of straight lines?

(ii) Find the equation of the bisectors of the angles between the lines $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$. 2+6

(b) Reduce the following equation to its canonical form : $16x^2 - 24xy + 9y^2 - 104x - 172y + 44 = 0$. Find the nature of the conic and find the equation of its axis. 6+1+1

2. If the two conics $\frac{l_1}{r} = 1 - e_1 \cos\theta$ and $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$ touch one another, then show that

$$l_1^2\left(1 - e_2^2\right) + l_2^2\left(1 - e_1^2\right) = 2l_1 l_2 \left(1 - e_1 e_2 \cos\alpha\right).$$ 6

3. Find the locus of the poles of the normal chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 6

4. The products of the lengths of the tangents drawn from a point P to the parabola $y^2 = 4ax$ is equal to the product of the focal distance of P and the latus rectum. Prove that the locus of P is the parabola $y^2 = 4a(x + a)$. 6

5. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from the origin, then show that $f^4 - g^4 = c(bf^2 - ag^2)$. 6

Group - B

(Marks : 15)

Answer ***question no.* 6** and ***any two*** questions from the rest.

6. Find the equation of the plane through the point (1, 2, 3) and perpendicular to the planes $2x + 3y - 4z = 9$ and $x + 2y + 2z = 5$. 3

Or,

Find the distance of the point (3, 8, 2) from the straight line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-2}{3}$ measured parallel to the plane $3x + 2y - 2z + 15 = 0$. 3

7. A variable plane has intercepts on the co-ordinate axes, the sum of whose squares is a constant k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(x^2 + y^2 + z^2\right)^2 \left(x^{-2} + y^{-2} + z^{-2}\right) = k^2 .$$ 6

8. A variable line, intersects the lines $y = 0, z = c$; $x = 0, z = -c$ and is parallel to the plane $lx + my + nz = p$. Prove that the surface generated by it is

$$lx(z - c) + my(z + c) + n(z^2 - c^2) = 0.$$ 6

9. Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$, $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Find also the equations and the points of intersection in which it meets the lines. 6

10. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the axes in A, B, C. Prove that the planes through the axes and the internal bisectors of the angles of the triangle ABC pass through the line,

$$\frac{x}{a\sqrt{b^2+c^2}} = \frac{y}{b\sqrt{c^2+a^2}} = \frac{z}{c\sqrt{a^2+b^2}}.$$ 6

Group - C

(Marks : 15)

Answer ***any three*** questions.

11. Forces $\vec{P}, \vec{Q}$ act at O and have a resultant $\vec{R}$. If any transversal cut their lines of action at A, B and C respectively, prove by vector method that $\frac{P}{OA} + \frac{Q}{OB} = \frac{R}{OC}$. 5

12. Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$, where α and β are both acute angles and $\alpha > \beta$. 5

13. Prove that for any three vectors $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$, $\left[\vec{\alpha}\times\vec{\beta}\ \ \vec{\beta}\times\vec{\gamma}\ \ \vec{\gamma}\times\vec{\alpha}\right]=\left[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}\right]^2$. 5

14. Obtain the vector equation of the straight line through the points $\hat{i}-2\hat{j}+\hat{k}$ and $3\hat{k}-2\hat{j}$. Show that this line intersects the plane passing through the origin and the points $4\hat{j}$ and $2\hat{i}+\hat{k}$ at a point given by $\frac{1}{5}\left(6\hat{i}-10\hat{j}+3\hat{k}\right)$. 5

15. (a) If $\vec{a},\vec{b},\vec{c}$ be three non-coplanar, non-zero vectors, then show that any vector $\vec{d}$ can be expressed as

$$\vec{d}=\frac{[\vec{b}\,\vec{c}\,\vec{d}\,]\vec{a}+[\vec{c}\,\vec{a}\,\vec{d}\,]\vec{b}+[\vec{a}\,\vec{b}\,\vec{d}\,]\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}\,]}.$$

(b) Find the moment about the point $\left(\hat{i}+2\hat{j}-\hat{k}\right)$ of a force represented by $\left(3\hat{i}+\hat{k}\right)$ acting through the point $\left(2\hat{i}-\hat{j}+3\hat{k}\right)$. 3+2